Point Vortex Model of Deflected Wakes of Oscillating Airfoils

X. He

China Aerodynamics Research and Development Center, Mianyang, Sichuan, 621000, PR China

I. Gursul

Department of Mechanical Engineering, University of Bath, Bath, BA2 7AY, UK

I. Introduction

There is considerable interest in biologically inspired flapping wings for micro air vehicle applications [1]. Thrust generation and its relation to the wake flow for the plunging and pitching airfoils have been reviewed by Platzer *et al.* [2]. The most important parameter used in the study of oscillating airfoils is the Strouhal number based on the amplitude, $St_A = fA/U_\infty$, where f is the oscillation frequency, A is the peak-to-peak amplitude of the trailing-edge, and U_∞ the freestream velocity. Experimental and computational studies have found that at sufficiently high Strouhal numbers, a transition occurs in the wake vortex pattern. Clockwise and counter-clockwise trailing-edge vortices may begin to shed in pairs and propagate at an angle to the freestream direction as sketched in the inset of Figure 1. This symmetry breaking process was observed experimentally by many investigators for pitching and plunging airfoils [3-10]. Numerical simulations also revealed this phenomenon [3,11-13]. Observations indicate that deflected wakes form once the trailing-edge vortices reach sufficient strength at high Strouhal numbers. The direction of the deflected wake can be up or down, depending on the initial conditions. Although deflected wakes were not observed for a low aspect ratio wing [14], there is probably a transition to the 2D airfoil case with increasing aspect ratio. There is potential for deflected wakes to be used for substantial lift enhancement of micro air vehicles [15] as well as other flow control applications.

For zero mean angle of attack, the two possible wake modes are mirror images of each other. In this paper it is referred to as symmetric bifurcation. When the mean angle of attack is nonzero, two different flow fields are possible if the mean angle of attack is equal or less than the stall angle [9]. Asymmetric bifurcation refers to nonzero mean angle of attack, for which the two possible wake modes are not mirror images. The two possible modes, based on the measurements in [9], are schematically sketched in Figure 1 as the mean angle of attack is varied.

The deflection angles might be both negative (downward deflected wake) for the two possible flow fields [9] with increasing angle of attack. For angles of attack larger than the stall angle, there is only one possible flow field (a downward deflected wake). It has been observed that a nonzero angle of attack causes an asymmetry in the circulation of the clockwise and counter-clockwise vortices. Cleaver *et al*. [9] suggested that a vortex asymmetry parameter can be defined, which represents dimensionless angular velocity of a vortex pair. The experimental data suggest that the upward-deflected wake is not possible once the asymmetry parameter exceeds a critical value.

It is believed that trailing-edge vortices alone are responsible for deflected wakes, although leading-edge vortices are also present in the experiments. Jones *et al*. [4] used an unsteady inviscid panel code, and simulated deflected jets on a plunging two-dimensional airfoil. This suggests that the formation of deflected jets is an inviscid phenomenon and leading-edge separation/vortex is not an essential part. Both experiments and simulations show that the direction of the deflected wake is determined by the sign of the *starting vortex* for the symmetric bifurcation. For nonzero mean angle of attack, the direction of the deflected wake is determined by the angle of attack and the sign of the *starting vortex*. It is clear that wake deflection phenomenon strongly depends on the initial conditions. Motivated by these observations, we construct a simple model in this paper. We neglect the vortex formation process, type of airfoil motion (pitching or plunging), and viscosity (initially). We release point vortices into the flow periodically and follow their motion. This can be justified as the roll-up of the vorticity into concentrated vortices is completed fairly quickly in a short streamwise distance from the airfoil at high Strouhal numbers. While the formation of vortex dipoles has been used in previous studies [7] to explain the deflected wakes, our model differs in two aspects: 1) previous models treat static vortex configurations and look for a criterion for symmetry breaking, whereas we study a starting (transient) flow model to explain the formation of wake deflection. 2) The criterion suggested in [7] requires detailed flow information such as the circulation, spacing and orientation of the vortices. In our simple model, these parameters are not needed once the Strouhal number and oscillation amplitude are given.

II. Vortex Model and Initial Conditions

The initial conditions are shown in Figure 2. The vortices are released from $y = \pm A/2$ at $x = 0$ which corresponds to the streamwise location of the trailing-edge of the airfoil. The vortices of the same sign are released with a frequency of f. Then the vortices move under the influence of the other vortices and freestream velocity. We follow the motion of the vortices in the flow field. The dimensionless equations of the motion for the vortex located at (X_k, Y_k) are:

$$\frac{fA}{U_\infty}\left[\frac{dX_k}{d\tau} - i\frac{dY_k}{d\tau}\right] = \frac{\Gamma}{U_\infty A}\frac{1}{2\pi i}\sum_{j=1}^{N}\frac{\Gamma_j/\Gamma}{Z_k - Z_j} + 1. \qquad (1)$$

where $Z = X+iY$ is the complex variable, τ dimensionless time, and $|\Gamma_j| = \Gamma$ for the zero mean angle of attack (symmetric bifurcation) case [16]. This equation suggests that two dimensionless numbers govern the motion: the Strouhal number based on the amplitude $St_A = fA/U_\infty$ as expected, and the dimensionless circulation $\Gamma / U_\infty A$.

In our simple model, we neglect how the vortices form and shed, and we release point vortices periodically. We do not try to model unsteady aerodynamics of an oscillating airfoil, but are interested in a simple model of the wake dynamics. As we are not concerned with how the airfoil generates the discrete vortices (i.e., we do not have an "airfoil"), we did not enforce the Kelvin theorem for the symmetric bifurcation case. The bound vortex is likely to have negligible effect as it is located far upstream (on the order of 10A) for small amplitude oscillations.

III. Symmetric Bifurcation

In this section we discuss the zero mean angle of attack case only. For given values of $\Gamma / U_\infty A$ and St_A, Equation [1] was integrated with the initial conditions shown in Figure 2. We used the *Matlab* (*ode45*) function to solve the ordinary differential equation. The calculations were continued for up to $t/T = 100$, where $T = 1/f$ is the period of the airfoil oscillations. In most cases, the deflection angle θ (measured at the origin using the youngest two vortices with the same sign) reaches a stable value after 5-10 cycles [16]. (This transient period seems to increase with the mean angle of attack). For the initial conditions shown in Figure 2, there are only two vortices (one vortex dipole). Therefore, this vortex dipole moves at an angle to the freestream due to its self-induced velocity whose magnitude and angle depend on the circulation and spacing between the vortices (hence the Strouhal number).

Figure 3 shows the comparison of the predicted and measured deflection angles as a function of Strouhal number for a NACA0012 airfoil plunging at a fixed amplitude [9]. It is seen that our simple numerical model reasonably predicts the experimental data. We also made comparisons with the experiments on pitching airfoils reported in [7]. They were presented in [16] and showed that, although the trend was captured well, the agreement was not as good as for plunging airfoils. This could be due to the weaker vortices (smaller circulation) in the pitching airfoil experiments.

Figure 4 shows the variation of predicted deflection angle as a function of Strouhal number for various values of the dimensionless circulation. The deflection angle increases with increasing circulation. It is seen that the deflection angle initially increases with increasing Strouhal number for a given circulation. The maximum deflection angle is observed in the range of St_A = 0.3-0.6, which is similar to the range of all experimental observations available in the literature. Subsequent decrease of the deflection angle with increasing Strouhal number is not unexpected. As the Strouhal number increases, the direction of the self-induced velocity approaches the direction of the freestream. This behaviour at large Strouhal numbers is due to the change in the geometry of the youngest dipole. With increasing Strouhal number, the wavelength becomes smaller (vortices move closer), approaching a synthetic jet in the limiting case of infinitely large Strouhal number. In Reference [16], the initial deflection angle was also calculated from the first vortex couple shown in Figure 2 as a function of the Strouhal number and circulation parameter, which shows similar trends to those in Figure 4, but it is generally greater.

It can be shown that the multiplication of the two parameters, $(\Gamma / U_\infty A)St_A$ represents the ratio of magnitude of the induced velocity to the freestream velocity [16]. Both the experimental data on plunging and pitching airfoils for various oscillation amplitudes [4,7-9,17] and the model reveal reasonable collapse, and show increasing deflection angle with increasing values of this parameter [16]. One may also consider the effect of three-dimensionality on the deflected wakes of oscillating airfoils by introducing a reduction in the circulation of the vortices as a function of their ages, which yields reasonable agreement with the experiments [16].

IV. Asymmetric Bifurcation

For zero mean angle of attack, the initial conditions shown in Figure 2 will always lead to an upward deflected wake (positive θ). If the initial conditions are mirror imaged, we obtain a downward deflected wake (negative θ). The asymmetric bifurcation is possible at nonzero mean angles of attack. Both modes (possible vortex streets) can be downward deflected, depending on the angle of attack. In order to simulate dual flow fields (two possible wake modes), we used two initial conditions as follows: the initial vortex configuration shown in Figure 2 is defined as Mode A and the mirror image is defined as Mode B. Then, in order to model the deflected wakes, we follow the same methodology with two additional features: (i) a bound vortex representing the mean circulation of the airfoil is placed on the x-axis at a negative x-location corresponding to the quarter-chord location of the virtual airfoil. The lumped vortex in this case has a clockwise circulation. (ii) Circulations of the clockwise and counter-clockwise vortices are taken from the measurements of Cleaver *et al*. [9], which are unequal due to the nonzero mean angle of attack.

Figure 5 shows the deflection angles for the two modes that were measured experimentally as well as the predictions, which are in very good agreement. Increasing bias towards downward-deflected wakes with increasing angle of attack as well as the lack of dual flow fields at the largest angle of attack (α = 15°) are captured remarkably. This figure also shows the calculated deflection angles for equal strength vortices (average of the experimentally measured circulations), which are not even able to predict the change of the upward-deflected wakes to the downward-deflected wakes with increasing angle of attack. We also carried out calculations for the experimentally measured circulations (nonzero asymmetry parameter) but with no bound vortex for $\alpha = 15°$. Figure 5 shows that the effect of the bound vortex (mean circulation) is small in near-wake dynamics. This is not surprising as the bound vortex is far upstream for small amplitude oscillations (x/A on the order of 10). The single important parameter appears to be the difference in the strength of the vortices.

To understand the effect of the difference in the circulation of the vortices better, we present the location of the vortices in Figure 6 for $\alpha = 15°$ and Mode A, at different times following the start of the motion. The left column shows the calculated results for experimentally measured circulations (counter-clockwise vortex is stronger) and the right column presents the simulation results for the equal circulations (average of experimentally measured circulations). We also performed simulations for equal strength vortices with the maximum measured circulation (not shown here), but found a very small difference. Figure 6 shows that, the newest clockwise vortex is located further downstream and downwards with increasing time (left column), whereas the vortex couple translates with a constant straight-line speed for the equal strength case (right column). The locations of the three youngest vortices at times when a new counter-clockwise vortex is generated at $x = 0$ are shown on the same plot and compared with the equal strength case in Figure 7. For unequal strength counter-rotating vortices, the vortices rotate around each other. As a result, the youngest clockwise vortex, which is the weaker one, moves further downstream compared to the equal vortices. This brings the youngest clockwise vortex closer to the counter-clockwise vortex from the previous cycle, and they form a new couple. At the same time, the counter-clockwise vortex from the previous cycle induces a velocity on the youngest clockwise vortex in the downward and downstream direction. This process of changing partner causes the wake to deflect downward as time goes on. The formation of the new couple and switching partner with an older vortex from the previous cycle is the main mechanism of changing from an initially upward deflected wake to a downward deflected wake.

In Reference [16], we reported further numerical experiments in which we vary the difference in the circulation of the vortices. As shown in Figure 5 for zero asymmetry there is no switch of the wake for α = 15°, whereas for the asymmetry parameter $\Delta\Gamma/U_{\infty}A$ =3.15 (corresponding to the experiments) we are able to reproduce the experimental observations of the wake deflection. When the difference in the circulation of the vortices is varied, we find that once the critical value $\Delta\Gamma/U_{\infty}A \approx 0.5$ is exceeded, an upward deflected wake is not possible. This compares well with the experimental observations of the critical asymmetry parameter, $\Delta\Gamma/U_{\infty}A \approx 0.8$, derived from the measurements reported in [9].

V. Conclusions

It is well known from experiments and computational simulations that, at sufficiently high Strouhal numbers, oscillating airfoils exhibit deflected near-wake vortex patterns. We model this phenomenon as a starting wake flow and release point vortices into the freestream by neglecting the vortex formation process. Comparison with the experiments on pitching and plunging airfoils reveals that the model is able to reasonably predict the deflection of the wakes. When the first vortex couple is released, their self-induced velocity is directed at an angle to the freestream, and its magnitude depends on the Strouhal number and normalized circulation. The starting flow forms an upward or downward deflected wake, depending on the initial vortex configuration. Dual wake modes of the asymmetric bifurcation observed in the experiments for nonzero mean angle of attack was simulated by adding a bound lumped vortex to represent the mean circulation of the airfoil and by using the experimentally measured circulations of the vortices. The results are in good agreement with the experiments, and predict the increasing bias towards downward-deflected wakes with increasing angles of attack. We show that the difference in the strength of the vortices is the most important parameter. The predicted critical asymmetry parameter (difference in the strength of vortices) beyond which an upward deflected wake is not possible is in good agreement with the experiments. The mechanism of switching from an initially upward-deflected wake to a final downward-deflected wake involves the weaker vortex changing partner and forming a new couple with the stronger vortex from the previous cycle.

Acknowledgements

The authors would like to acknowledge the support of the China Scholarship Council and the China Aerodynamics Research and Development Center.

References

[1] Shyy, W., Berg, M. and Ljungqvist, D. "Flapping and flexible wings for biological and micro air vehicles". *Progress in Aerospace Sciences*. Vol. 35, No. 5, 1999, pp. 455–505.

[2] Platzer, M.F., Jones, K.D., Young, J. and Lai, J.C.S. "Flapping-wing aerodynamics: Progress and challenges". *AIAA J*. vol. 46, No. 9, 2008, pp. 2136–2149.

[3] Bratt, J.B. "Flow patterns in the wake of an oscillating airfoil". *Aeronautical Research Council*, Report No. R & M 2773, 1950.

[4] Jones, K.D., Dohring, C.M. and Platzer, M.F. "Experimental and computational investigation of the Knoller-Betz effect". *AIAA Journal*. Vol. 36, 1998, pp. 1240-1246.

[5] Heathcote, S. and Gursul, I. "Jet switching phenomenon for a periodically plunging airfoil". *Physics of Fluids*. Vol. 19, No. 22, 2007, 027104.

[6] Godoy-Diana, R., Aider, J.L. and Wesfreid, J.E. "Transitions in the wake of a flapping foil". *Phys. Rev. E*. vol. 77, 2008, 016308.

[7] Godoy-Diana, R., Marais, C., Aider, J.L. and Wesfreid, J.E. "A model for the symmetry breaking of the reverse Benard-von Karman vortex street produced by a flapping foil", *Journal of Fluid Mechanics*. Vol. 622, 2009, pp. 23-32.

[8] von Ellenrieder, K.D. and Pothos, S. "PIV measurements of the asymmetric wake of a two dimensional heaving hydrofoil", *Experiments in Fluids*. Vol. 44, No. 5, 2008, pp. 733-745.

[9] Cleaver, D.J., Wang, Z. and Gursul, I., "Bifurcating flows of plunging airfoils at high Strouhal numbers", *Journal of Fluid Mechanics*. Vol. 708, 2012, pp. 349-376.

[10] Cleaver, D.J., Wang, Z. and Gursul, I. "Investigation of mechanisms of high lift for a flat-plate airfoil undergoing small-amplitude plunging oscillations", *AIAA Journal*, vol. 51, no. 4, 2013, pp. 968-980.

[11] Lewin, G.C. and Haj-Hariri, H. "Modelling thrust generation of a two-dimensional heaving airfoil in a viscous flow", *Journal of Fluid Mechanics*. Vol. 492, 2003, pp. 339-362.

[12] Emblemsvag, J.E., Suzuki, R. and Candler, G. "Numerical simulation of flapping micro air vehicles", AIAA Paper 2002-3197, *32nd AIAA Fluid Dynamics Meeting*, 2002, St. Louis, MO; US.

[13] Liang, C.L., Ou, K., Premasuthan, S., Jameson, A. and Wang, Z.J. "High-order accurate simulations of unsteady flow past plunging and pitching airfoils". *Computers & Fluids*, vol. 40, no. 1, 2011, pp. 236-248.

[14] Calderon, D.E., Cleaver, D.J., Gursul, I. and Wang, Z. 2014 "On the absence of asymmetric wakes for periodically plunging finite wings", *Physics of Fluids*, vol. 26, 2014, 071907; doi: 10.1063/1.4891256.

[15] Gursul, I., Cleaver, D.J. and Wang, Z. "Control of low Reynolds number flows by means of fluid-structure interactions", *Progress in Aerospace Sciences*, vol. 64, 2014, pp. 17-55.

[16] He, X. and Gursul, I., "A Simple Model of Asymmetric Wakes for Periodically Oscillating Airfoils", AIAA-2016-1604, 54th AIAA Aerospace Sciences Meeting, 4-8 January, 2016, San Diego, CA.

[17] Bohl, D. G. & Koochesfahani, M. M. "MTV measurements of the vortical field in the wake of an airfoil oscillating at high reduced frequency". *J. Fluid Mech.* Vol. 620, 2009, pp. 63–88.

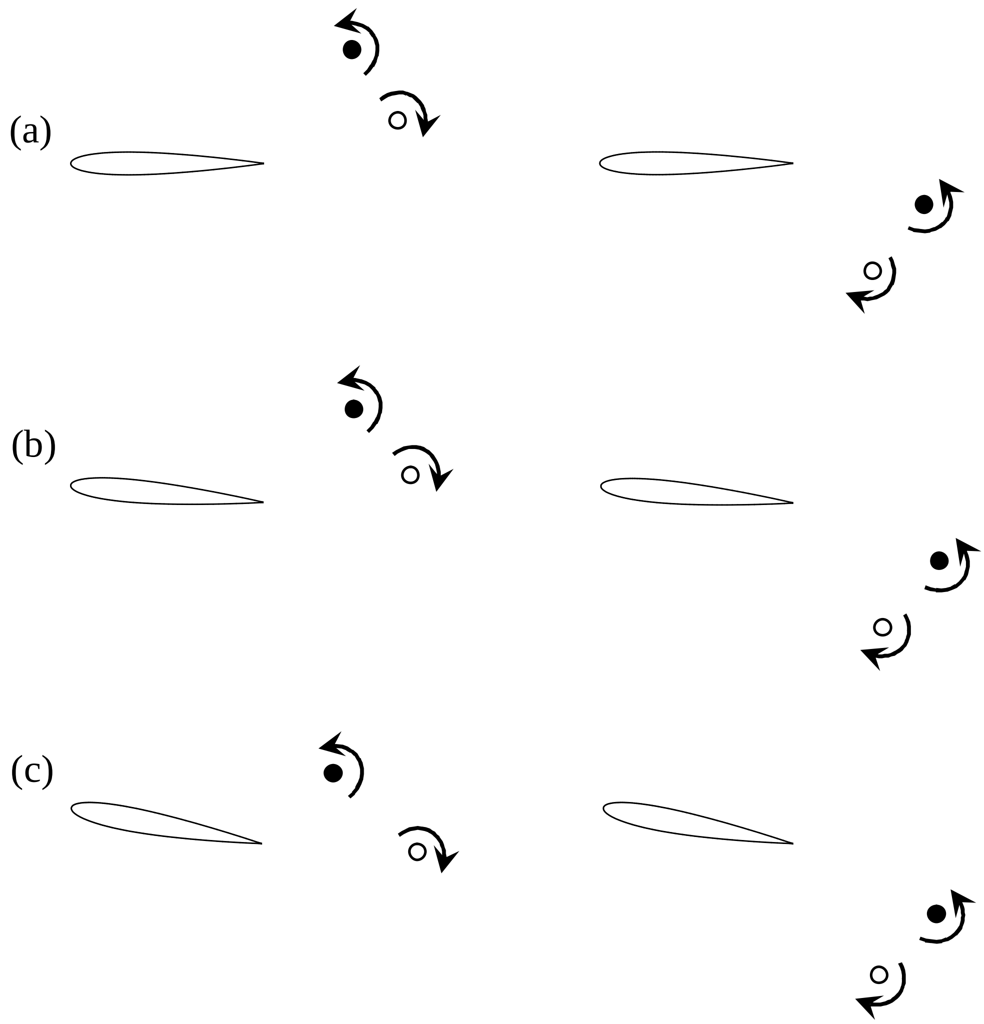


Figure 1: Schematic effect of mean angle of attack on two possible vortex streets for a) α=0°, b) α=5°, c) α=10°, based on the measurements in [9].

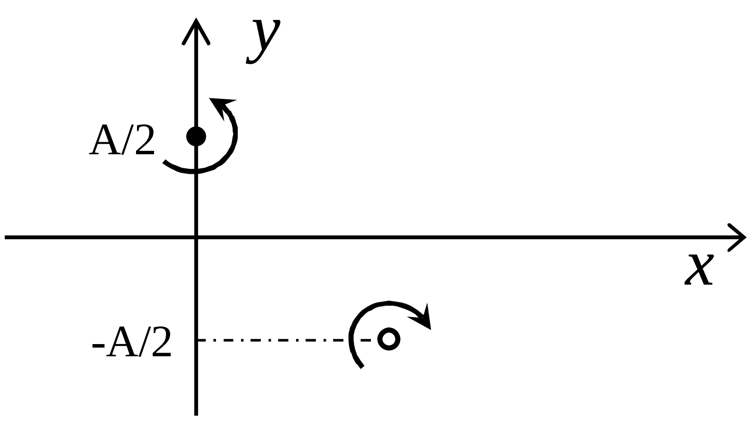


Figure 2: Initial vortex configuration at t = 0. The free stream velocity acts in the direction of positive x-axis.

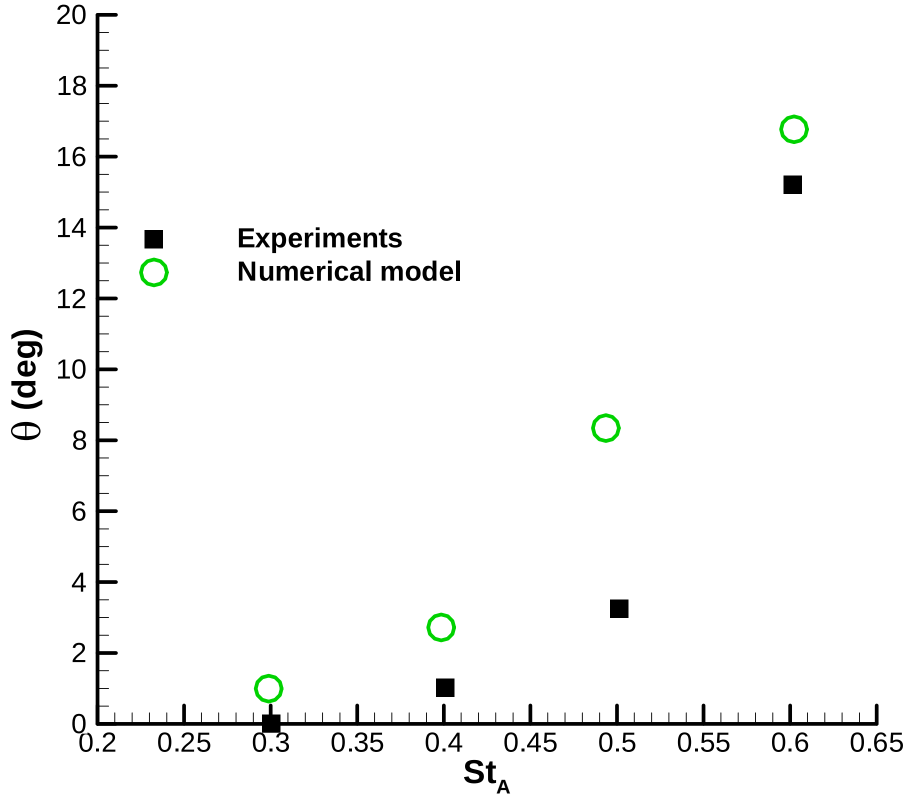


Figure 3: Variation of measured and predicted deflection angles with Strouhal number for plunging airfoil. Experimental data taken from Cleaver *et al.* (2012).

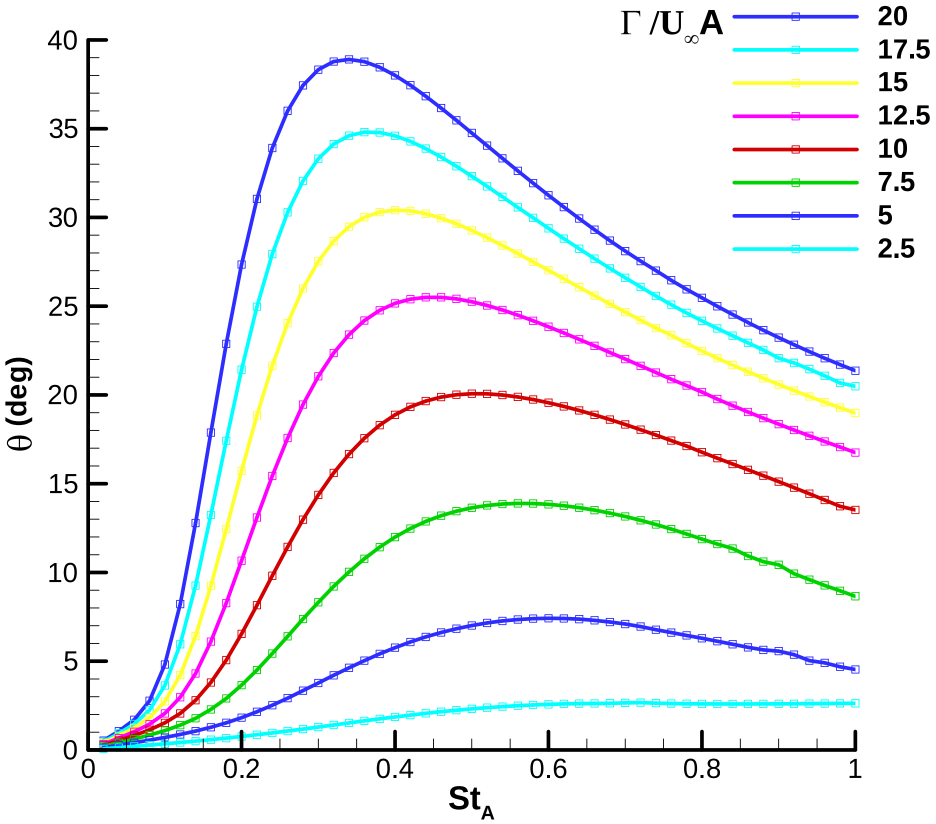


Figure 4: Variation of predicted deflection angle with Strouhal number for various values of circulation parameter.

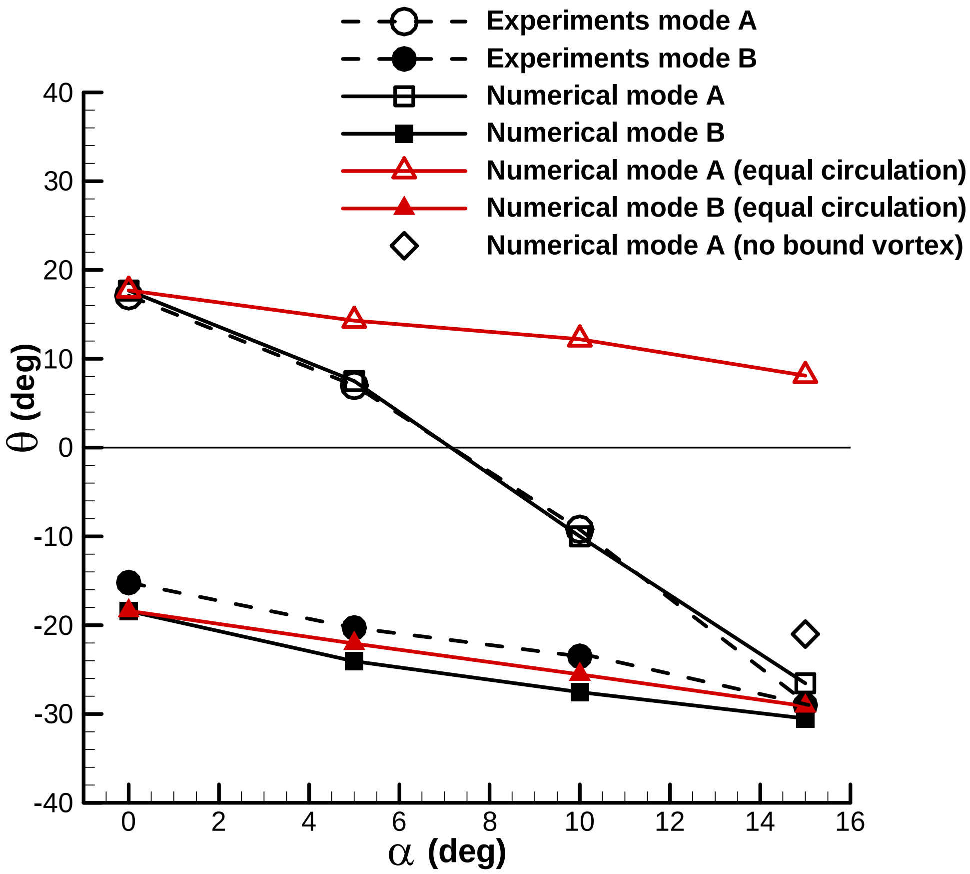


Figure 5: Effect of mean angle of attack for plunging airfoil for Mode A and Mode B, $St_A = 0.6$.

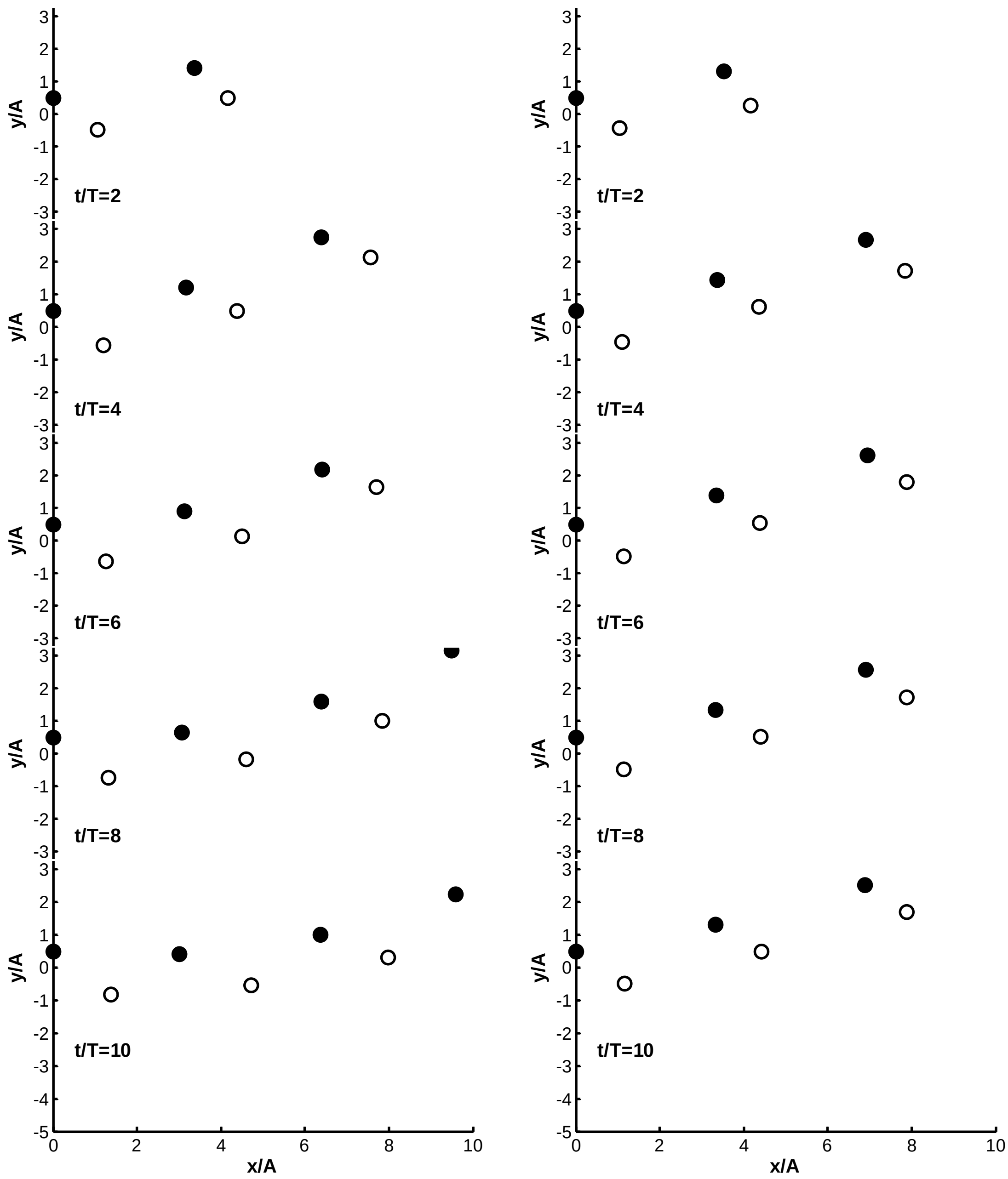


Figure 6: Location of vortices at each time a new counter-clockwise vortex is generated at x = 0. Unequal vortex circulations: counter-clockwise $|\Gamma_+|$ = 10.55 and clockwise $|\Gamma_-|$ =7.40 (left column); and equal vortex circulation: $|\Gamma_+|$ = $|\Gamma_-|$ = 9.00 (right column).

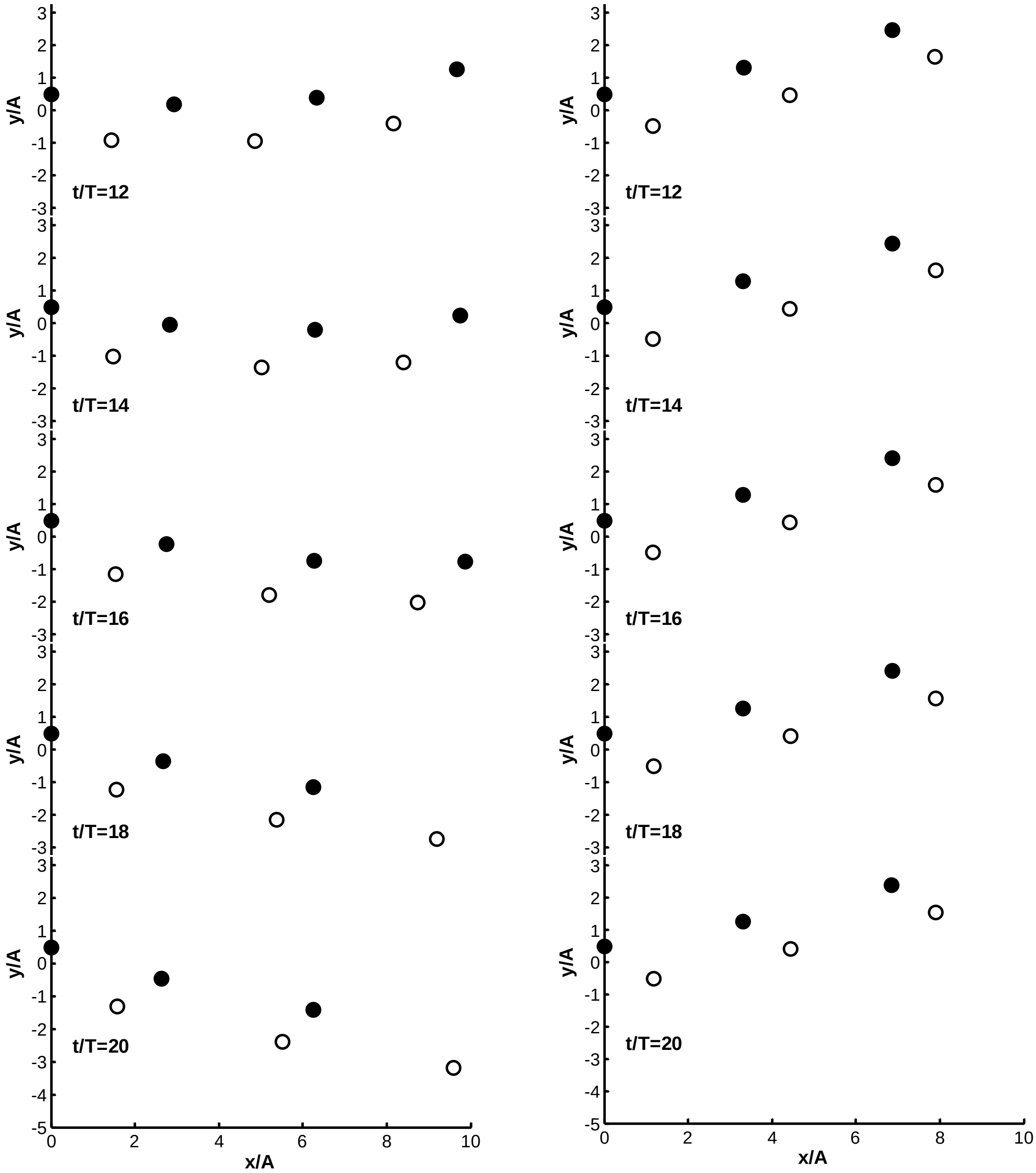
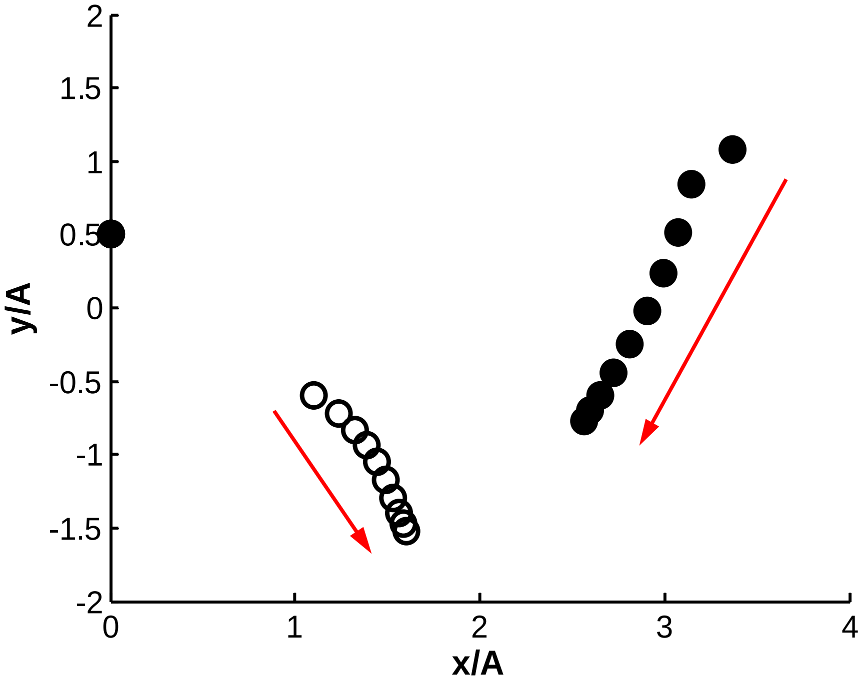


Figure 6 continued.

(a)



(b)

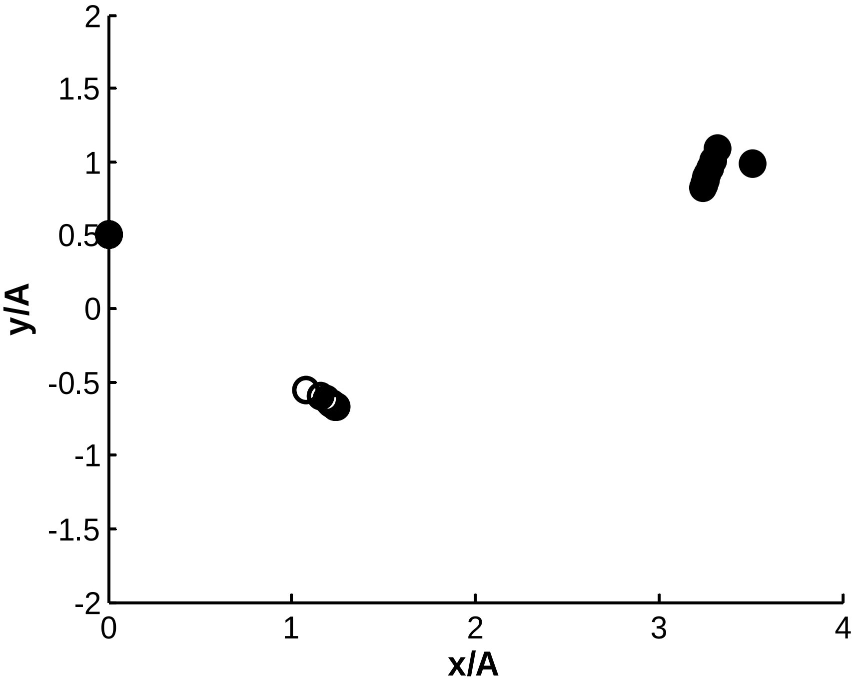


Figure 7: Locations of the three newest vortices at times when a new counter-clockwise vortex is generated at $x = 0$ for (a) unequal circulations (top) and (b) equal circulations (bottom) as in the caption of Figure 6.